SPECIALIST MATHEMATICS

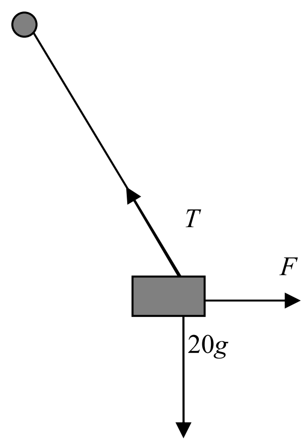
Units 3 & 4 – Written examination 1

2007 Trial Examination

SOLUTIONS

Question 1

a.



A1

b. $\sin\theta = \dfrac{0.5}{2} = \dfrac{1}{4}$ so by triangle or Pythagoras, $\cos\theta = \dfrac{\sqrt{15}}{4}$.

A1

$20g = T\cos\theta = \dfrac{T\sqrt{15}}{4}$ (downward)

A1

$F = T\sin\theta = \dfrac{T}{4}$ (horizontally)

Eliminating T and rationalizing gives $F = \dfrac{4g\sqrt{15}}{3}$

A1

Question 2

$$y = \int \frac{e^{2x}dx}{(e^{2x})^2 + 3}.$$

A1

Substitution is $u = e^{2x}$ so $\frac{du}{dx} = 2e^{2x}$.

A1

$$y = \frac{1}{2}\int \frac{du}{u^2+3} = \frac{1}{2\sqrt{3}}\int \frac{\sqrt{3}\,du}{u^2+3}$$

M1, A1

$$= \frac{1}{2\sqrt{3}}\tan^{-1}\frac{e^{2x}}{\sqrt{3}} + c$$

If $y = 0$ when $x = 0$, $c = -\frac{1}{2\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}} = \frac{-\pi}{12\sqrt{3}}$

$$y = \frac{1}{2\sqrt{3}}\tan^{-1}\frac{e^{2x}}{\sqrt{3}} - \frac{\pi}{12\sqrt{3}}$$ A1

Question 3

a. $-2 - 2\sqrt{3}i = 4\,cis\frac{-2\pi}{3}$

A1

b. $z - 2 + i = \pm 2\,cis\frac{-\pi}{3} = \pm(1 - \sqrt{3}i)$ M1

$z = 3 - (1+\sqrt{3})i$ or $1 - (1-\sqrt{3})i$ A1 + A1

Question 4

a. Using implicit differentiation (product rule on LHS) gives

$2x + 4xy' + 4y + 2y' = 0$ M1

Regrouping $(4x+2)y' = -2x - 4y$

So $\frac{dy}{dx} = -\frac{2x+4y}{4x+2}$

$\frac{dy}{dx} = -\frac{x+2y}{2x+1}$ A1

b. If $x = 1$, $1 + 4y + 2y = -11$ so $y = -2$. A1

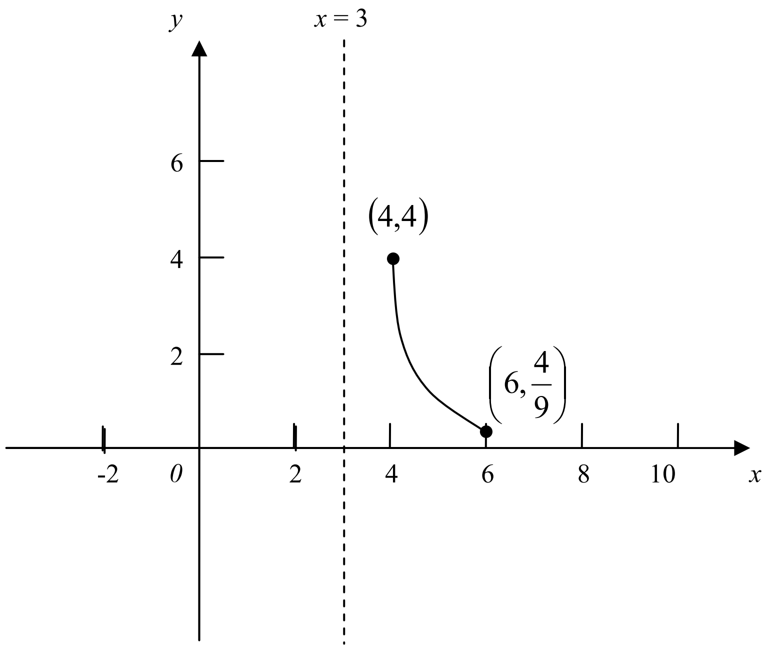
Substituting in derivative $\frac{dy}{dx} = -\frac{1-4}{2+1} = 1$. A1

Question 5

a. $x = t + 3$, so $t = x - 3$. $y = \dfrac{4}{t^2} = \dfrac{4}{(x-3)^2}$, A1

Domain: $1 \le t \le 3$ becomes $1 \le x - 3 \le 3$ or $4 \le x \le 6$. A1

b.



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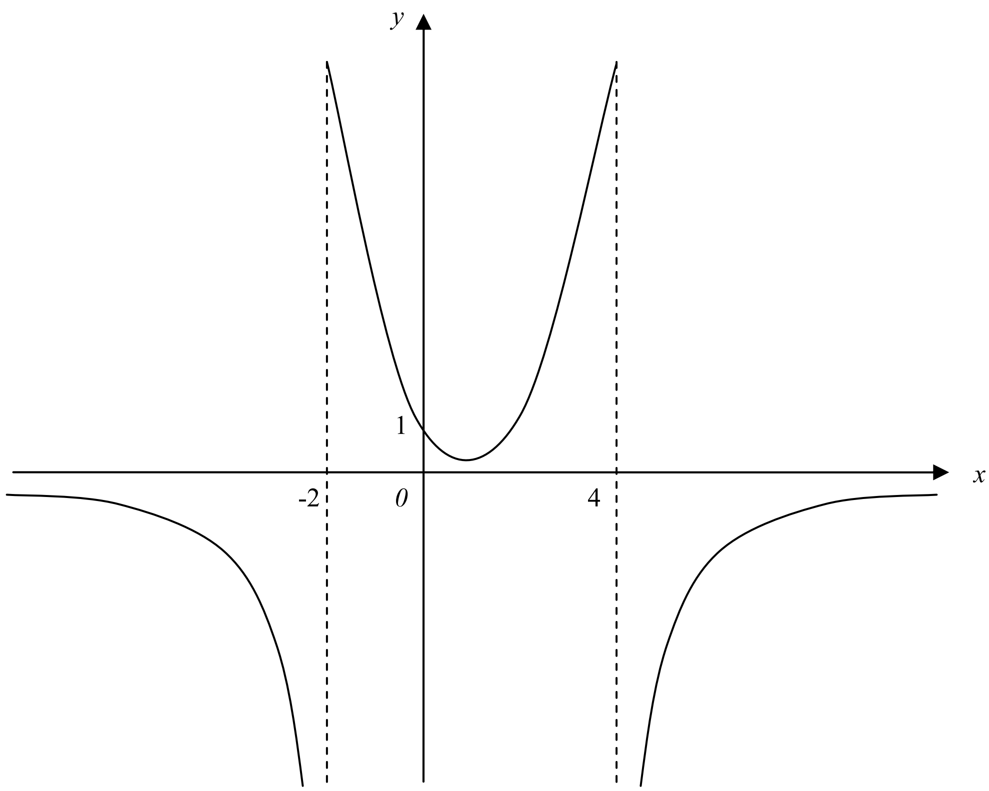
Shape A1

End points A1

Question 6

a. Asymptotes $x = -2,\ x = 4.$ A1

Turning point $\left(1, \frac{8}{9}\right)$, intercept (0,1). A1



A1

b. $$\int_0^2 \frac{8dx}{(4-x)(x+2)} = \frac{8}{6}\int_0^2 \left(\frac{1}{4-x}+\frac{1}{x+2}\right)dx$$ M1

$$=[\frac{4}{3}\log_e(x+2)-\frac{4}{3}\log_e(4-x)]_0^2$$ A1

$$=\frac{4}{3}\left(\log_e 4-\log_e 2-\log_e 2+\log_e 4\right)$$

M1 A1

$$=\frac{4}{3}\log_e 4=\frac{8}{3}\log_e 2$$

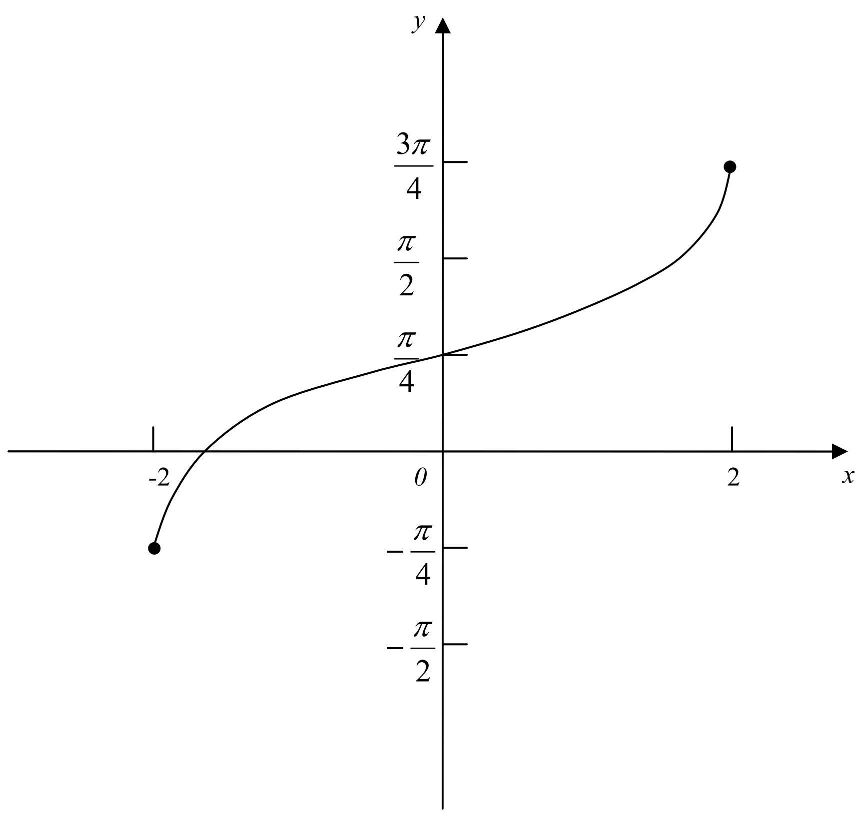
Question 7

a. $y = \int \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\frac{x}{2} + c$ A1

If $x = 0,\ y = \frac{\pi}{4}$ so $c = \frac{\pi}{4}$.

$y = \sin^{-1}\frac{x}{2} + \frac{\pi}{4}$ A1

b.



Shape A1
End points A1

Question 8

$u = \sin 3x$, so $\dfrac{du}{dx} = 3\cos 3x$, $dx = \dfrac{du}{3\cos 3x}$ A1

$$y = \int \frac{dx}{\cos 3x} = \frac{1}{3}\int \frac{du}{\cos^2 3x} = \frac{1}{3}\int \frac{du}{1-u^2}$$ M1

$$= \frac{1}{6}\int \left(\frac{1}{1-u} + \frac{1}{1+u}\right) du = \frac{1}{6}\log_e \frac{1+u}{1-u} + c,$$ M1

$$= \frac{1}{6}\log_e \frac{1+\sin 3x}{1-\sin 3x} + c. \text{ So } a = \frac{1}{6}, \quad f(x) = \frac{1+\sin 3x}{1-\sin 3x}$$

A1

Question 9

a. $V = \pi \int_0^a \cos^2 \frac{x}{4} \, dx$.

A1

b. $V = \dfrac{\pi}{2}\displaystyle\int_0^a \left(1 + \cos\frac{x}{2}\right) dx = \dfrac{\pi}{2}\left[x + 2\sin\frac{x}{2}\right]_0^a$

M1

$$= \frac{\pi}{2}\left(a + 2\sin\frac{a}{2}\right)$$

A1

c. We require $\dfrac{\pi}{2}\left(\dfrac{\pi}{3} + 1\right) = \dfrac{\pi}{2}\left(a + 2\sin\dfrac{a}{2}\right)$.

If $a = \dfrac{\pi}{3}$, it is easy to check that $2\sin\dfrac{a}{2} = 2\sin\dfrac{\pi}{6} = 2 \times \dfrac{1}{2} = 1$.

A1